University of Mannheim
School of Social Sciences
Math Refresher for Political Science, Fall 2025
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Solutions: Probability Theory II

1. Consider the p.d.f. $f(x) = 2x$ for $0 \leq x \leq 1$.

   (a) Calculate the c.d.f. of $f(x)$.

   $$F(x) = x^2$$

   (b) Is $f(x)$ a proper p.d.f.?

   $$\begin{aligned} \int_0^1 F(x) &= 1 \\ 1^2 - 0^2 &= 1 \end{aligned}$$

   $f(x)$ is a proper p.d.f.

2. Consider the c.d.f. $G(x) = \frac{1}{9}x^2$ for $0 \leq x \leq 3$.

   (a) Calculate the p.d.f. of $G(x)$, $g(x)$.

   $$g(x) = \frac{2}{9}x$$

   (b) Is $g(x)$ a proper p.d.f.?

   $$\begin{aligned} \int_0^3 G(x) &= 1 \\ \frac{1}{9} \times 3^2 - \frac{1}{9} \times 0^2 &= 1 \end{aligned}$$

   $g(x)$ is a proper p.d.f.

3. Consider the p.d.f. $h(x) = \frac{4}{3}(1 - x^3)$ for $0 < x < 1$. Determine

   (a) $\Pr(X < \frac{1}{2})$.

   $$H(x) = -\frac{1}{3}x(x^3 - 4)$$
   $$H\left(\frac{1}{2}\right) \approx 0.65$$

(b) $\Pr(X > \frac{1}{3})$.

$$1 - H\left(\frac{1}{3}\right) \approx 0.56$$

(c) $\Pr(\frac{1}{4} < X < \frac{3}{4})$.

$$H\left(\frac{3}{4}\right) - H\left(\frac{1}{4}\right) \approx 0.56$$

4. Consider the p.d.f. $k(x) = cx^2$ for $1 \le x \le 2$. Determine

(a) Find the value of the constant c.

$$\begin{aligned} \int_1^2 k(x)dx &= 1 \\ \frac{1}{3}cx^3\Big|_1^2 &= 1 \\ c &= \frac{3}{7} \end{aligned}$$

(b) Find $\Pr(X > \frac{3}{2})$.

$$K(x) = \frac{1}{7}x^3$$
$$1 - K\left(\frac{3}{2}\right) \approx 0.52$$